Comment on “Pionic decay of a possible d' dibaryon and the short-range NN interaction”

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We comment on calculations of the width of the d' resonance within framework of quark shell models.

In a recent paper [1] by I. Obukhovsky, K. Itonaga, Georg Wagner, A. Buchmann and A. Faessler the decay of the d' resonance is calculated in a microscopic quark shell model. This resonance has been suggested to explain peculiarities in the forward angle cross section of the pionic double charge exchange (DCX) on nuclei at low energies [2]. Quantum numbers of the resonance, $T(J^P) = 0(0^-)$, imply that the decay $d' \to$NNπ is dominated by s–waves between the outgoing particles. In general the strong interaction d'NNπ vertex contains two independent invariant amplitudes (see ref. [3]). However at low energies (like e.g. in the decay $d' \to$NNπ) only one amplitude survives.*

As calculated recently in ref. [1], the decay amplitude has to go as $\mathbf{k}^2$, where $\mathbf{k}$ is the 3-momentum of the outgoing pion in the d' rest frame. This result for the s–wave decay $d' \to$NNπ is hardly possible to advocate since $\mathbf{k}^2$–behaviour would be a feature of a d–wave (or double p–wave within the 3-body system of outgoing particles) process.

On the other hand in ref. [3] it has been already shown that the d' decay amplitude has to be proportional to the pion 4-momentum k_μ since this amplitude has to satisfy Adler self-consistency condition. Thus the leading term in the amplitude is the one containing the time component k_0 of the 4-vector k_μ. This result can be corroborated within a model similar to that used by the authors of ref. [1].

In ref. [1] it is assumed that the d' wave function corresponds to the s^5p^1 configuration, while the outgoing $6q$ system, carrying quantum numbers $J^P = 0^+$, $T = 1$, can be a mixture of a number of configurations. Let us take into account one of them, namely s^6 configuration. It will be easy to see, that the result holds true for the other configurations as well.

In the model considered in ref. [1] the NNπ decay of the d' is due to the presence of the $qq\pi$–vertex, giving rise to the transition

$$(s^5p^1) \to (s^6) + \pi \ , \tag{1}$$

followed by the subsequent fall-apart of the outgoing $6q$–system into NN pair in the $^1S_0$ state.

The $qq\pi$–vertex can be written in the pseudo-scalar form, $\bar\psi\gamma_5\vec\tau\psi\vec\pi$, or preferably in the pseudo-vector form [4]:

$$f_{qq\pi}\bar\psi\gamma_\mu\gamma_5\vec\tau\psi\cdot\partial_\mu\vec\pi \tag{2}$$

Thus it is explicitly seen that a vertex of a pion emission in any elementary act is proportional to the pion 4-momentum. This does not exclude, of course, an extra dependence of the whole amplitude of a physics process on the pion momentum. The coupling constant $f_{qq\pi}$ has the dimension of length, and can be fixed so as to reproduce the NNπ coupling constant.

The probability amplitude of the transition (1) is proportional to the amplitude of the transition of a quark from p to s shell, $q_p \to q_s + \pi$, accompanied by the pion emission. The latter equals

$$f_{qq\pi}\int \bar\psi_s(\mathbf{r})\gamma_\mu\gamma_5\vec\tau\psi_p(\mathbf{r})\ k_\mu\ \vec\pi\ e^{i\mathbf{kr}}d\mathbf{r}\ , \tag{3}$$

if the pion is described by a plane wave, and ψ_s and ψ_s are the wave functions (bispinors), describing quarks on s and p shells, respectively.

Wave function of a fermion in a state with the definite total angular momentum j, its projection m, and parity P reads as:

$$\psi_{jm}(\mathbf{r}) = \begin{pmatrix} f(r)\Omega_{jlm}(\mathbf{n}) \\ (-1)^{\frac{1+l-l'}{2}}g(r)\Omega_{jl'm}(\mathbf{n}) \end{pmatrix}\ , \tag{4}$$

where Ω_{jlm} are spherical spinors depending on $\mathbf{n} = \mathbf{r}/r$, $r = |\mathbf{r}|$ (see e.g. [5]). $l = j \pm 1/2$, and $l' = 2j - l$. For a given j the states with $l = j - 1/2$ and $l = j + 1/2$ have different parity. The spherical spinor $\Omega_{jl'm}$ can be expressed through Ω_{jlm} as:

$$\Omega_{jl'm} = i^{l-l'}\ (\boldsymbol{\sigma}\mathbf{n})\Omega_{jlm}\ . \tag{5}$$

For a quark on $s_{1/2}$ and $p_{1/2}$ shells the corresponding bispinors (4) look particularly simple:

$$\psi_{s_{1/2}} = \begin{pmatrix} f_0(r)\varphi \\ g_0(r)(\boldsymbol{\sigma}\mathbf{n})\varphi \end{pmatrix}\ , \tag{6}$$

and

*This is easy to see since in that case the NN–pair is in the $^1S_0$–state ($J^P = 0^+$), hence the 4-body vertex contains 3 spinless “particles”: 0^- (π), 0^- (d'), and 0^+ (dinucleon).

$$\psi_{p_{1/2}} = \begin{pmatrix} f_1(r)(\boldsymbol{\sigma}\mathbf{n})\chi \\ g_1(r)\chi \end{pmatrix} . \quad (7)$$

Here φ and χ are nonrelativistic 2-component spinors which do not depend on $\mathbf{n}$.

It is easy to see that the leading contribution in eq.(3) is the one proportional to the total energy of the pion, k_0. †

For $\psi_{s,p}$ given by eqs.(6) and (7) the expression (3) can be presented in a simple form:

$$f_{qq\pi}(\varphi^+\chi)\left[-k_0\int(g_0^* f_1 + f_0^* g_1)e^{i\mathbf{kr}}d\mathbf{r} \right.$$
$$\left. + \int(f_0^* f_1 + g_0^* g_1)(\mathbf{kn})e^{i\mathbf{kr}}d\mathbf{r} \right. , \quad (8)$$

in which integration over angles is trivial.

Further details depend of course on the potential used to find solution for $\psi_{s,p}$, and in particular on the Lorentz structure of the potential.

For the $qq\pi$ vertex written in the pseudo-scalar form the amplitude of the transition (1) is proportional to

$$(\varphi^+\chi)\int(g_0^* f_1 - f_0^* g_1)e^{i\mathbf{kr}}d\mathbf{r} . \quad (9)$$

Thus we conclude that for both types of the $qq\pi$ coupling (pseudo-vector and pseudo-scalar) the amplitude of the s–wave decay $d' \to$NNπ does not vanish at $\mathbf{k} \to 0$, as distinct from the statement in ref. [1], according to which the amplitude is proportional to $\mathbf{k}^2$. The result of ref. [1] is therefore appears to be erroneous.

The amplitude of the s–wave decay $d' \to$NNπ is strongly modified by the NN final state interaction, as had been shown in our paper from 1993 (see ref. [3]), in particular if the decay process is due to a point-like interaction responsible for a $q\bar{q}$–pair creation.‡ This effect is known to give preference to small NN invariant masses, and might be a crucial for the experimental searches for the d'. Thus e.g. by applying a cut in the NN invariant masses one expects an enhancement of the signal-to-background ratio. It is this procedure which has been applied to sense the d' contribution in the experiment on double pion production, $pp \to pp\pi^-\pi^+$, performed at ITEP (Moscow) [6].

In ref. [3] we considered the simplest case to take into account NN FSI:
1) point like d'NNπ vertex, and
2) Yamagichi wave function for the NN in the continuum.

Decay of the d' into $pp\pi^-$ is also affected by Coulomb effects (see pion spectra in the decays $d' \to nn\pi^+$ and $d' \to pp\pi^-$ in ref. [3]), leading to some 10 – 15 % difference in the decay rates $d' \to nn\pi^+$ and $d' \to pp\pi^-$.

The effects of the NN FSI in the decay $d' \to pp\pi^-$ can be taken into account (see e.g. [7]) by multiplying the differential probability of the decay by

$$F_{pp}(p) = F_C(pa_c) \quad (10)$$

$$\times \left| 1 + \frac{\beta + ip}{-a_s^{-1} + \frac{1}{2}r_0 p^2 - \frac{2}{a_c}h(pa_c) - ipF_C(pa_c)} \right|^2$$

where $p \equiv |\mathbf{p}|$, and $\mathbf{p}$ is the 3-momentum of either proton in the pp c.o.m. Functions $F_C(x)$ and $h(x)$ are given by:

$$F_C(x) = \frac{\frac{2\pi}{x}}{e^{\frac{2\pi}{x}} - 1} \quad (11)$$

and

$$h(x) = \frac{1}{x^2}\sum_{n=1}^{\infty}\frac{1}{n(n^2 + x^{-2})} - \gamma + \ln(x) , \quad (12)$$

where $\gamma = 0.577...$ is the Euler constant. a_s is the pp scattering length, r_0 – effective range, and a_c is the Bohr radius for the pp subsystem; $\beta \approx 230$ MeV is the parameter of the Yamaguchi potential. Numerical result obtained with eq.(10) is only 20–30% different from the "exact" one (see e.g. recent preprint [8]) obtained from the solution of the wave equation with "Coulomb + Yamaguchi potential" [9].

Thus we see that the dependence of the d'NNπ amplitude on the pion momentum, k_μ, in combination with the FSI effects appears to be extremely important for any spectra in the d' decay. An extra factor $\mathbf{k}^4$ in the amplitude squared would have led to even stronger enhancement of the high energy side of the pion spectrum (corresponding to small pp invariant masses). However this is not so, and k–dependence of the invariant amplitude squared is given by the product $k_0^2 F_{pp}(p)$, where k_0 is the total energy of the pion in the d' rest frame.

Potential models similar to that used in ref. [1] should lead to a vanishingly small radiative decay rate $d' \to d\gamma$. This decay mode is of special interest since γd is the only elementary process where d' should manifest itself as a

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† Another simple way to prove this statement is to consider 3-body vertex of the d' decay into two spinless "particles", pion and di-nucleon [NN]. Let $\vec{\pi}$, $\vec{\Phi}$ and Ψ be the operators, annihilating pion, [NN] and d', respectively. (Here the sign vector stands for the isospin degrees of freedom). The only way to construct the Lorentz invariant vertex describing emission of a pion by the axial current is: $(\vec{\Phi}^+ \overleftrightarrow{\partial}_\mu \Psi)(\partial_\mu\vec{\pi})$. The amplitude of the decay $d' \to$[NN]π is then proportional to the scalar product of the 4-vectors $k(P_{[NN]} + P_{d'})$, where $P_{d'} = P_{[NN]} + k$. This scalar product equals $M_{d'}^2 - M_{[NN]}^2$; for a small energy release, $\Delta E = M_{d'} - M_{[NN]}$ (which is the case in the d' decay), it equals $2\Delta E \cdot M_{[NN]} \approx 4M_N k_0$ to a good accuracy since $\Delta E \approx k_0$, while extra terms proportional to powers of $\mathbf{k}^2$ are very small.

‡ The numerical result for the enhancement caused by the NN FSI is of course a model dependent.

Breit–Wigner pole in s–channel. The nature of suppression of the isoscalar E1 transition $d' \to d\gamma$ is the same as in nuclear physics where E1 transitions with $\Delta T = 0$ between nuclei with $N = Z$ are known to be forbidden (L.Radicati, 1952, see e.g. [7]). The same holds true not only for potential quark models, but also for any quark cluster model where (effective) mass of a quark cluster is proportional to the number of quarks in the cluster. An example, demonstrating this statement was considered in ref. [10], where it was assumed that the d' represents an orbitally excited state composed of a diquark ud ($S = T = 0$, $colour = \bar{3}$) with angular momentum $l = 1$ relative to a four-quark cluster $uudd$ ($S = 1$, $T = 0$, $colour = 3$). The corresponding E1 amplitude of the transition $d' \to d\gamma$ is then proportional to $e_2/m_2 - e_4/m_4$, where e_n and m_n are electric charge and mass of the n–quark cluster, respectively. The E1 amplitude vanishes if $m_4 = 2m_2$ because $e_4 = 2e_2$ $(= 2/3)$.

For the configuration s^5p^1 considered in ref. [1] sum of the E1 amplitudes corresponding to the two possible splittings $u - uuddd$ and $d - uuudd$ also vanishes if $m_5 = 5m_q$.

Let us note that in the potential model used in ref. [1] the d' mass appears to be too large, while in the flux-tube model with anomalously light diquark ud ($S = T = 0$) the mass of the 0^- isoscalar $6q$–state is rather close to the one needed for the dibaryon explanation of the peculiarities in the pionic DCX. Simultaneously the existence of light diquarks makes ineffective the suppression of the E1 transitions discussed above.

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